

# Efficient fax transmission of halftone images

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**Abstract.** *The efficient encoding and transmission of information for facsimile communication relies on redundancy in the scanned pixels. Halftone images, especially those rendered by high-quality dispersed dot techniques, are "busy" with alternative black and white pixels and shorter run lengths as compared to text information. Because of this, it is desirable to increase the redundancy and decrease the entropy of those images for efficient encoding and transmission. We propose a novel technique whereby both transmitting and receiving fax devices have in memory a halftone screen such as the "blue noise mask" (BNM). The BNM is a halftone screen that produces a visually appealing dispersed dot pattern with an unstructured, isotropic pattern. When both the transmitting and receiving fax devices have the same halftone screen in ROM, the problem of halftone image encoding can be reduced to that of transmitting the mean gray value of blocks, or subimages, followed by a sparse halftone error image with increased redundancy and run-lengths compared to the original halftone. Examples show that by using the proposed technique, image entropy can be reduced to 0.2 bits/pixel, and typical run-lengths can increase by a factor of 5. The increase in image quality, combined with increased transmission speed, could add considerably to the utilization and acceptance of halftone fax images.*

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## 1 Introduction

The efficient transmission of fax data takes advantage of the redundancy in printed text. Long run-lengths of black or white pixels are represented using modified Huffman and

modified READ codes, and line-to-line redundancy further reduces the number of bits per page required to reconstruct the facsimile.<sup>1,2</sup> Redundancy-reduction codings are standardized by the CCITT as the coding schemes for Group 3 and Group 4 fax equipment. This fax coding scheme uses a two-dimensional line-by-line coding method in which the position of each changing picture element on the current coding line is coded with respect to the position of a corresponding reference element situated on the reference line located immediately above the coding line. After the coding line has been coded, it becomes the reference line for the next coding line.<sup>1,2</sup>

A problem occurs when gray-scale images, instead of text, are scanned.<sup>3</sup> Since most fax printers are capable of only black or white output, the image must first be halftoned. The halftone image is typically composed of a mosaic of black and white pixels, and is not well suited for conventional redundancy-reduction coding. The compression of binary images has been studied from different points of view including predictive encoding, arithmetic coding, and progressive hierarchical coding.<sup>4-6</sup> No one method has yet gained widespread use.

Many different halftoning techniques exist, but the most visually pleasing of these produce a fine dispersed, unstructured, isotropic pattern of black and white pixels known as "blue noise."<sup>7</sup> Until recently, the blue noise pattern was produced by a family of algorithms, known as "error diffusion" algorithms.<sup>8</sup> A disadvantage of this approach is that each new image requires new computations to render the halftone pattern. A recent development, the "blue noise mask" (BNM) is a halftone screen (not an algorithm) that

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can be used to render a halftone rapidly with the desired dispersed dot, unstructured pattern of black and white pixels.<sup>9</sup> Because a fixed, unique halftone screen exists and can be reproduced in the multiple transmitter/receivers, the BNM can be utilized as an effective encoder/decoder of images. Other halftone screens can also be used in the fax transmission scheme, as described below.

## 2 The ToneFac Algorithm

In this discussion we assume that both transmitting and receiving devices have the same built-in halftone screen, such as the BNM, Bayer's, or a clustered dot screen.<sup>8</sup> We also assume that a gray-scale image has been digitized to  $M \times N$  pixels at  $B$  bits per pixel.

The ToneFac (halftone fax compression) algorithm begins with halftone rendering of the image.

### 2.1 Transmission Steps

**Step 1:** Halftone the gray image  $g(i, j)$  using the halftone screen,  $b(i, j)$  (both  $B$ -bit,  $M \times N$  arrays) using the simple comparison rule to produce the halftone image  $h(i, j)$ :

$$\forall g(i, j) \geq b(i, j) \Rightarrow h(i, j) = 1(\text{white}) ;$$

$$\forall g(i, j) < b(i, j) \Rightarrow h(i, j) = 0(\text{black}) .$$

**Step 2:** Choose a block size  $L \times K$  (smaller than  $M \times N$ ) to subdivide the gray image. Calculate the local mean gray image  $\bar{g}(J)$  for each  $L \times K$  subregion or block:

$$\bar{g}(J) = \frac{1}{L \times K} \sum_{i=1}^L \sum_{j=1}^K g(i, j) ,$$

where the summation is taken to be over the  $J$ 'th subregion.

Note that the optimal size of the subregion,  $L \times K$ , can be determined empirically based on the total time required to perform steps 5 and 6 below, or on consideration of the power spectrum or correlation length of the image. Furthermore, the median operator can be used instead of the mean, with some further improvement in the error image but at the expense of greater computation in this step.

**Step 3:** Create the halftone image of the mean block level,  $\bar{h}(J)$ , using the same halftone screen:

$$\forall i, j \in J; \bar{g}(J) \geq b(i, j) \Rightarrow \bar{h}(i, j) = 1 ;$$

$$\bar{g}(J) < b(i, j) \Rightarrow \bar{h}(i, j) = 0 .$$

The step produces a "blocky" halftone image,  $\bar{h}(i, j)$ , which effectively captures the halftone pattern of the low-frequency components of the image. Furthermore, a receiver with the same halftone screen in ROM can rapidly reconstruct the  $\bar{h}(i, j)$  simply by receiving the  $\bar{g}(J)$  in a prearranged order.

**Step 4:** Create the error image  $e(i, j)$ , which is the difference between the desired halftone image  $h(i, j)$  and the block mean halftone image  $\bar{h}(i, j)$ :

$$e(i, j) = [h(i, j) - \bar{h}(i, j)] \bmod 2 .$$

Note that the smaller the block size  $L \times K$ , the sparser the error image. On the other hand, extra time will be required to transmit the mean values of those additional blocks. Considering the error image, although  $e(i, j)$  can be  $+1, 0$ , or  $-1$ , only two states, i.e., 1 and 0, are necessary to represent "error" and "nonerror" states between the two binary images and this can be achieved by taking modulo 2. We will show that the desired halftone image  $h(i, j)$  can be reconstructed from the mean values  $\bar{g}(J)$  and  $e(i, j)$  at the receiver.

**Step 5:** Transmit the  $\bar{g}(J)$  in sequence for all  $J$ . There are  $N \times M/K \times L$  blocks (i.e.,  $J$  runs from 1 to  $N \times M/K \times L$ ). These mean values ( $B$  bits per block) can be encoded in any standard manner.

**Step 6:** Transmit  $e(i, j)$ . This error image is sparse since it is nonzero only in high-contrast and high-frequency regions of the image. Thus, it is more efficient for standard CCITT encoding schemes.

### 2.2 Receiver Steps

**Step 1:** Receive the  $\bar{g}(J)$  and produce the block halftone image:

$$\forall i, j \in J; \bar{g}(J) \geq b(i, j) \Rightarrow \bar{h}(i, j) = 1 ;$$

$$\bar{g}(J) < b(i, j) \Rightarrow \bar{h}(i, j) = 0 .$$

**Step 2:** Receive  $e(i, j)$  and produce the desired halftone image:

$$h(i, j) = [\bar{h}(i, j) + e(i, j)] \bmod 2 .$$

Once  $h(i, j)$  is available, it can be printed and/or stored in a conventional manner.

Transmission steps, along with scanning and receiver steps, can also be performed concurrently so that a minimal amount of time is required to fax a picture.

## 3 An Example of ToneFac

We have chosen to work with 8-bit images of sizes  $256 \times 256$  to  $512 \times 512$ . The "teddy bear" synthetic image (Fig. 1) was  $512 \times 512$ . The BNM is chosen to illustrate the algorithm because the BNM produces a high-quality dispersed dot halftone image.<sup>9</sup> A subdivision size of  $8 \times 8$  was chosen empirically to provide the minimum transmission time. The mean value of pixels in the gray-scale picture is calculated for each block, and a block halftoned image is generated. An error halftoned image is then obtained from taking the difference (modulo 2) between the block halftoned image and the original halftone image. Figure 1 shows the original, the block, and the error halftoned images of the "teddy bear" picture. Note that the error halftoned image is much less "busy" than the original halftoned image. The values for the mean of each block in the block halftoned image is transmitted, together with the entire error halftoned image. At the receiver side, the block halftone is reconstructed from the mean value of each block and the same blue noise mask. Adding the block halftoned image and the error halftoned image pixelwise, modulo 2, the original halftoned image is retrieved.

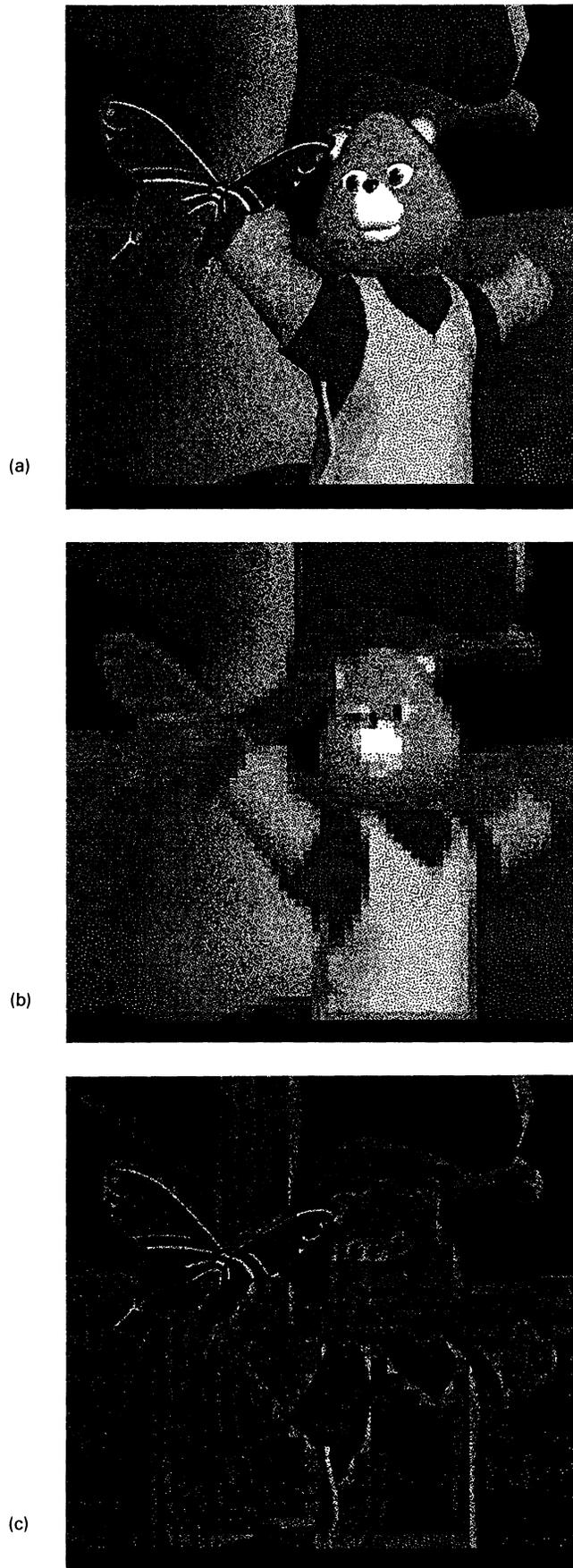


Fig. 1 (a) Original halftone, (b) block halftone, and (c) error halftone.

#### 4 Simulation Results

Figure 2 shows four gray-scale pictures used for computer simulations. The halftone image statistics are compared in two ways: the run-length and the line redundancy of the picture. The run-length statistics is further represented by the classical run-length entropy equation [Eq. (1)], described by Netravali, Mounts, and Beyer.<sup>10</sup>

$$E = \frac{E_w N_w + E_b N_b}{r_w N_w + r_b N_b}, \quad (1)$$

where

$E$  = entropy in bits/pel

$r_w, r_b$  = average white and black run-length (pels/run), respectively

$N_w, N_b$  = number of white and black runs, respectively.

Also,  $E_w$  and  $E_b$  are the entropy of the white and black run statistics (bits/run), respectively, and  $E_w$  is computed using the formula

$$E_w = - \sum_i \frac{n_i}{N} \log_2 \frac{n_i}{N}, \quad (2)$$

where  $n_i$  is the number of white runs of length  $i$ , and  $N$  is the total number of white runs. *Line correlation* (Table 1) is defined as the average percentage of pixels in each line that have the same value as the previous line above it.

Tables 1 and 2 show that the improvement in line-redundancy and run-length by the proposed technique is significant. It can also be noted that the proposed technique is most powerful for documents composed of low-frequency smooth pictures such as the "teddy bear" and "brain" images. The distribution of the line-redundancy is also shown in Fig. 3.

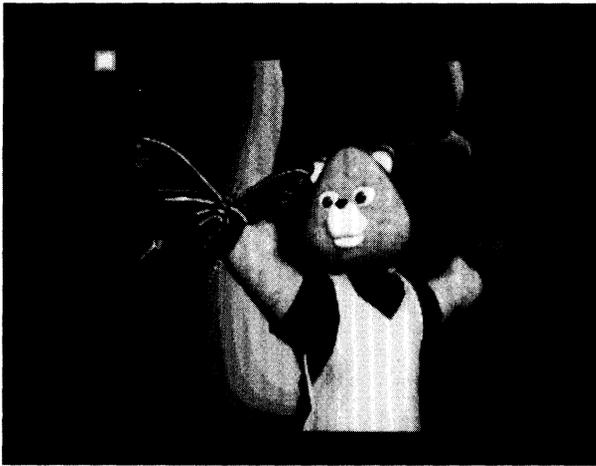
As mentioned above, when the median operator is used instead of the mean in step 2 of the transmitting algorithm, some improvement is gained in terms of decreased entropy. Using the median operator, the final entropy (bits/pixel) of "teddy bear" would be 0.19; "Lena," 0.32; "USC girl," 0.24; and "brain," 0.21.

#### 5 Use of Other Halftone Screens

It is possible to use other halftone screens in the ToneFac algorithm, and we have evaluated the use of  $8 \times 8$  pixel kernels of Bayer's dispersed dot dither and 45-deg clustered dot dither.<sup>8</sup> The entropies of the test pictures using different dithering are shown in Table 3. While it is possible to use other halftone screens, the BNM has the advantages of a nonperiodic, isotropic, unstructured, dispersed dot pattern that is visually pleasing.

#### 6 Fax Transmission Experiments

In a preliminary test, the original and error halftone images of "teddy bear" obtained from the BNM, clustered dot, and Bayer's dithering were sent between Sharp F0-420 and Sharp F0-330 facsimile devices. The images were enlarged by a factor of 4 (75-dpi equivalent print output) and the fax was set to fine resolution. The resulting transmission times are shown in Table 4. (Note that the transmission time of



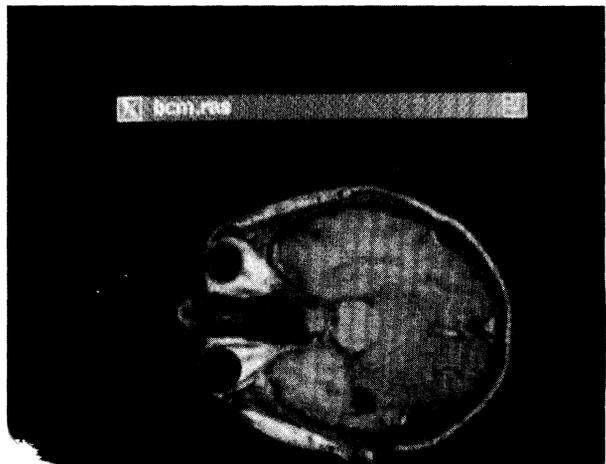
(a)



(b)



(c)



(d)

Fig. 2 Test pictures: (a) "teddy bear," (b) "Lena," (c) "USC girl," and (d) "brain."

Table 1 Comparison of line correlation.

Test File	mean line-to-line correlation (%)	
	Original	ToneFac
Teddy bear	67.90	93.41
Lena	48.63	88.14
uscgirl	61.21	91.96
brain	80.14	93.27

Table 2 Comparison of the run-length statistics.

Test File	Original		ToneFac	
	mean run-length (pixels)	entropy (bits/pixel)	mean run-length (pixels)	entropy (bits/pixel)
Teddy bear	3.17	0.76	15.82	0.22
Lena	1.93	0.94	8.49	0.34
uscgirl	2.60	0.75	12.72	0.25
brain	5.07	0.43	15.06	0.22

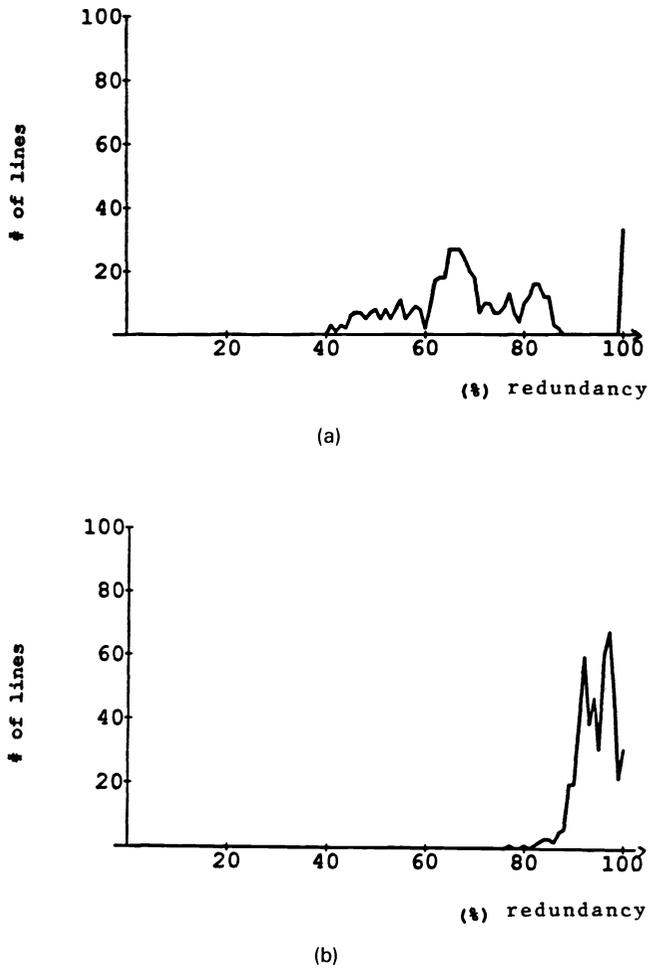


Fig. 3 Distribution of line redundancy for (a) original and (b) error image of "teddy bear."

a white paper of the same size is deducted to eliminate the overhead time required. Transmission time may vary with different fax devices.) The BNM error images required the shortest transmission times of the three halftone schemes. We have not added the transmission time for the mean values of the subregions because a variety of methods could be used to encode and transmit these values. Nevertheless, the time needed to transmit the mean values of the "teddy bear" with  $8 \times 8$  subregions would be roughly equivalent to transmitting 8 additional lines of a busy image.

### 7 Conclusion

A new technique for compression, transmission, and decompression of halftone images has been presented. The technique assumes that transmitting and receiving locations have the same halftone screen, such as the blue noise mask, which produces an unstructured and visually pleasing dispersed dot pattern. Images are encoded by subdividing the original image into subregions, or blocks. Transmission of the mean gray level of each block and of a sparse "error" halftone image is sufficient for reconstruction of the halftone at the receiver. Compared to the transmission of typically "busy" halftone images, the substitution of mean values and a sparse "error" image reduces entropy and increases average run-lengths by a factor of 4 to 5. This makes possible efficient transmission using conventional CCITT techniques and should increase the speed and quality of FAX halftones compared with current practices.

Compared with the recently proposed JBIG standards for bilevel images,<sup>6</sup> the ToneFac algorithm is simple to implement. The ToneFac, however, requires the use of an identical halftone screen at both the transmitter and receiver. Although the JBIG method would not require knowledge of the halftone screen, it is more complicated. Comparisons of the compression ratios obtained with each method on the same images is warranted.

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Table 3 Comparison of entropy using different halftone screens.

Test File	Error Image Entropy (bits/pixel)					
	BNM		Clustered Dot		Bayer's	
	original	ToneFac	original	ToneFac	original	ToneFac
Teddy Bear	0.75	0.22	0.40	0.19	0.35	0.19
Lena	0.94	0.32	0.56	0.33	0.46	0.32
uscgirl	0.75	0.25	0.42	0.23	0.33	0.20
brain	0.43	0.22	0.30	0.18	0.27	0.18

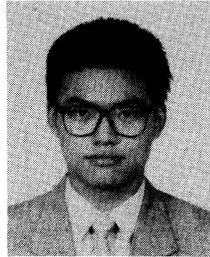
Table 4 Transmission time for the original and the error halftone images using different halftone screens.

	Transmission Time		
	BNM	Clustered Dot	Bayer's
Original	126	97	131
ToneFac	32	52	46

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**Kevin J. Parker:** Biography and photograph appear with the paper "Inverse halftoning" in this issue.



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